OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4732

Probability & Statistics 1

Thursday **9 JUNE 2005** Morning 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages and 3 blank pages.

Registered Charity Number: 1066969
[Turn over

1 (i) Calculate the value of Spearman's rank correlation coefficient between the two sets of rankings, A and B, shown in Table 1. [4]

A	1	2	3	4	5
B	4	1	3	2	5

Table 1

(ii) The value of Spearman's rank correlation coefficient between the set of rankings B and a third set of rankings, C, is known to be -1. Copy and complete Table 2 showing the set of rankings C. [2]

B	4	1	3	2	5
C					

Table 2

2 The probability that a certain sample of radioactive material emits an alpha-particle in one unit of time is 0.14. In one unit of time no more than one alpha-particle can be emitted. The number of units of time up to and including the first in which an alpha-particle is emitted is denoted by T.

(i) Find the value of

(a) $P(T = 5)$, [3]

(b) $P(T < 8)$. [3]

(ii) State the value of $E(T)$. [2]

3 In a supermarket the proportion of shoppers who buy washing powder is denoted by p. 16 shoppers are selected at random.

(i) Given that $p = 0.35$, use tables to find the probability that the number of shoppers who buy washing powder is

(a) at least 8, [3]

(b) between 4 and 9 inclusive. [2]

(ii) Given instead that $p = 0.38$, find the probability that the number of shoppers who buy washing powder is exactly 6. [3]

4 The table shows the latitude, x (in degrees correct to 3 significant figures), and the average rainfall y (in cm correct to 3 significant figures) of five European cities.

City	x	y
Berlin	52.5	58.2
Bucharest	44.4	58.7
Moscow	55.8	53.3
St Petersburg	60.0	47.8
Warsaw	52.3	56.6

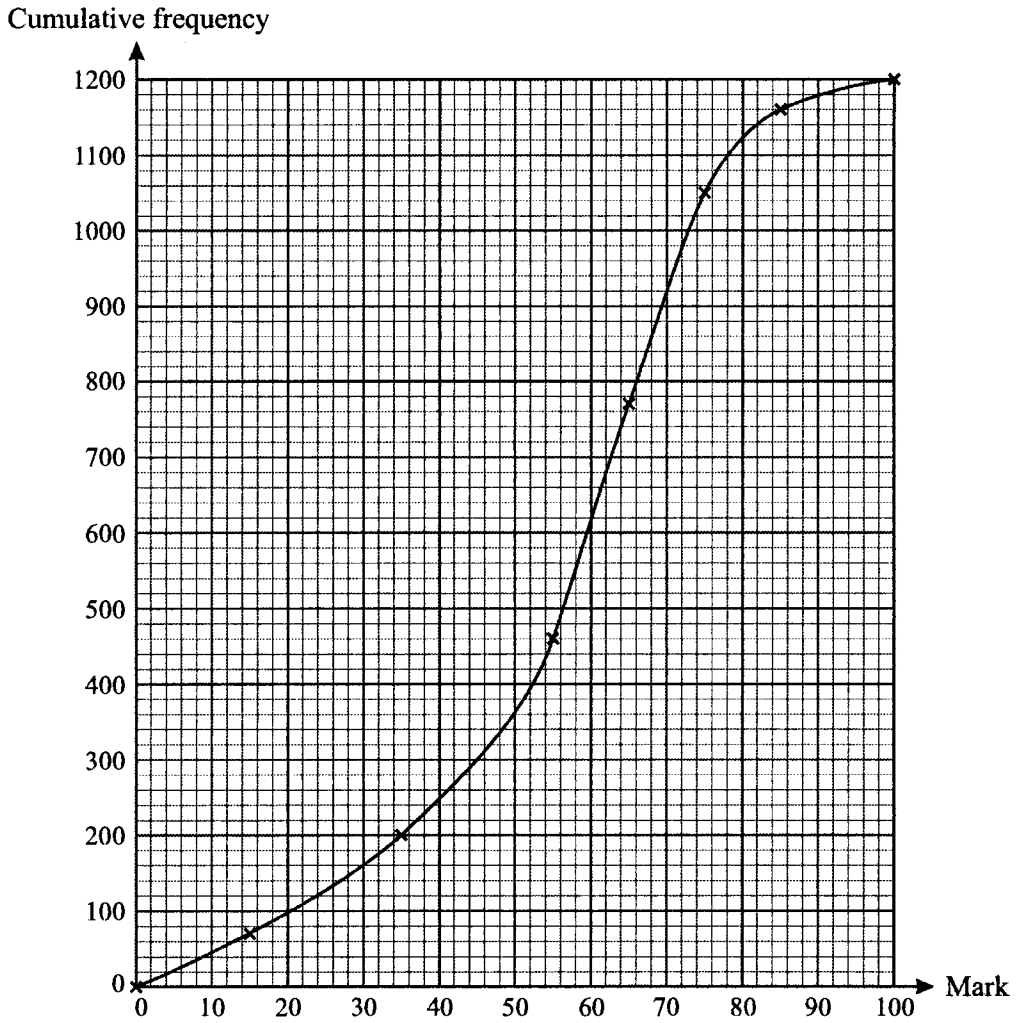
$[n = 5,\ \Sigma x = 265.0,\ \Sigma y = 274.6,\ \Sigma x^2 = 14\,176.54,\ \Sigma y^2 = 15\,162.22,\ \Sigma xy = 14\,464.10.]$

(i) Calculate the product moment correlation coefficient. [3]

(ii) The values of y in the table were in fact obtained from measurements in inches and converted into centimetres by multiplying by 2.54. State what effect it would have had on the value of the product moment correlation coefficient if it had been calculated using inches instead of centimetres. [1]

(iii) It is required to estimate the annual rainfall at Bergen, where $x = 60.4$. Calculate the equation of an appropriate line of regression, giving your answer in simplified form, and use it to find the required estimate. [5]

5 The examination marks obtained by 1200 candidates are illustrated on the cumulative frequency graph, where the data points are joined by a smooth curve.



Use the curve to estimate

(i) the interquartile range of the marks, [3]

(ii) x, if 40% of the candidates scored more than x marks, [3]

(iii) the number of candidates who scored more than 68 marks. [2]

Five of the candidates are selected at random, with replacement.

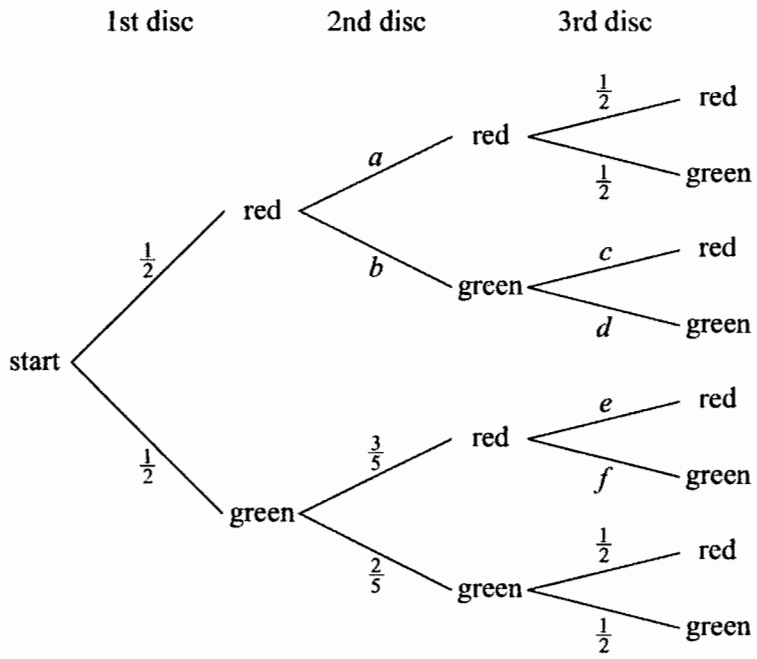
(iv) Estimate the probability that all five scored more than 68 marks. [3]

It is subsequently discovered that the candidates' marks in the range 35 to 55 were evenly distributed — that is, roughly equal numbers of candidates scored 35, 36, 37, ..., 55.

(v) What does this information suggest about the estimate of the interquartile range found in part **(i)**? [2]

6 Two bags contain coloured discs. At first, bag P contains 2 red discs and 2 green discs, and bag Q contains 3 red discs and 1 green disc. A disc is chosen at random from bag P, its colour is noted and it is placed in bag Q. A disc is then chosen at random from bag Q, its colour is noted and it is placed in bag P. A disc is then chosen at random from bag P.

The tree diagram shows the different combinations of three coloured discs chosen.



(i) Write down the values of a, b, c, d, e and f. [4]

The total number of red discs chosen, out of 3, is denoted by R. The table shows the probability distribution of R.

r	0	1	2	3
$P(R = r)$	$\frac{1}{10}$	k	$\frac{9}{20}$	$\frac{1}{5}$

(ii) Show how to obtain the value $P(R = 2) = \frac{9}{20}$. [3]

(iii) Find the value of k. [2]

(iv) Calculate the mean and variance of R. [5]

7 A committee of 7 people is to be chosen at random from 18 volunteers.

(i) In how many different ways can the committee be chosen? [2]

The 18 volunteers consist of 5 people from Gloucester, 6 from Hereford and 7 from Worcester. The committee is to be chosen randomly. Find the probability that the committee will

(ii) consist of 2 people from Gloucester, 2 people from Hereford and 3 people from Worcester, [4]

(iii) include exactly 5 people from Worcester, [4]

(iv) include at least 2 people from each of the three cities. [4]

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding

1 (i) Σd^2	M1		Subtr & squ 5 pairs & add
$= 14$	A1		
$1 - \frac{6 \times \textit{their}\ 14}{5 \times (25-1)}$	M1		dep 1st M1
$= 0.3$	A1	4	
			$S_{xy} = 48 - \frac{15 \times 15}{5}$ } { = 3 } $S_{xx} = 55 - \frac{15^2}{5}$ } M1 { = 10 } A1 $S_{yy} = 55 - \frac{15^2}{5}$ } { = 10 } their $\frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$ M1dep = 0.3 A1
(ii) Reverse rankings attempted	M1		3 correct
2 5 3 4 1	A1	2	T & I to make $\Sigma d^2 = 40$: 2 mks or 0 mks
	6		
2 (i) (a) Geo(0.14) stated in (a) or (b)	B1		or 0.86^nx0.14 or 0.14^nx0.86 in (a) or ≥ M1 in (b) or Geo(0.86) stated in (a) or (b)
$(0.86)^4$ x 0.14	M1		
$= 0.0766$ (3 sfs)	A1	3	No wking: 0.077: B1M1A0
(b) $1 - 0.86^7$	M2		$1 - 0.86^8$: M1
or $0.14 + 0.86$x$0.14 \ldots + 0.86^6$x0.14			$+8^{th}$ term (r = 7 or 0) or 1 missing term: M1
$= 0.652$ (3 sfs)	A1	3	
(ii) 1/0.14	M1		
$= {}^{50}/_{7}$ or 7.14 (3 sfs)	A1	2	
	8		
3 (i) (a) B(16, 0.35) stated	B1		Or implied by use of tables or 0.35^ax0.65^b (a+b = 16) in (a) or (b)
$1 - 0.8406$	M1		Allow 1 – 0.9329 or 0.0671 Or complete method using formula, P(r = 8-16 or 9-16) or 1-P(r = 0-7 or 0-8)
$= 0.159$ (3 sfs)	A1	3	
(b) 0.9771 – 0.1339	M1		Allow 0.9771 – 0.2892 Or complete method using formula (r = 4-9)
$= 0.843$ (3 sfs)	A1	2	
(ii) ${}^{16}C_6(0.38)^6(0.62)^{10}$	M2		Absent or incorr coeff : M1 or ${}^{16}C_6(0.38)^{10}(0.62)^6$: M1
$= 0.202$ (3 sfs)	A1	3	
	8		
4 (i) Correct subst in ≥ two S formulae	M1		Any correct version
$\frac{14464.1 - \frac{265 \times 274.6}{5}}{\sqrt{\left(14176.54 - \frac{265^2}{5}\right)\left(15162.22 - \frac{274.6^2}{5}\right)}}$	M1		or $\frac{14464.1 - 5 \times 53 \times 54.92}{\sqrt{(14176.54 - 5 \times 53^2)(15162.22 - 5 \times 54.92^2)}}$ or fully correct method with $(x - \bar{x})^2$ etc
$= -0.868$ (3 sfs)	A1	3	
(ii) No difference oe	B1	1	Or slightly diff or more acc because of rounding errors when mult by 2.54 oe Not just "more accurate"
(iii)Choose y on x stated	B1ind		or implied, eg by S_{xy}/S_{xx} or $y = ax + b$

$\dfrac{14464.1-\dfrac{265\times274.6}{5}}{14176.54-\dfrac{265^2}{5}}$ or -0.682	M1	If state x on y, but wking is y on x: B1 or their $\dfrac{-89.7}{131.54}$ seen or $\dfrac{14464.1-5\times53\times54.92}{14176.54-5\times53^2}$ or correct subst into a correct formula $\dfrac{S_{xy}}{S_{xx}}$
$y-{}^{274.6}/_{5}=$ (their -0.682)$(x-{}^{265}/_{5})$ $y=91(.1)-0.68(2)\,x$	M1ind A1	or $a={}^{274.6}/_{5}$ - (their -0.682) x ${}^{265}/_{5}$ Simplif to 3 terms. Coeffs to ≥ 2 sfs
49.9 (3sfs) or 50	5 A1	cao Use of x on y: equiv M mks as above
	9	
5 (i) Read at 300 or 300.25 and 900 or 900.75 44.5 to 45.5 and 69 to 69.9 IQR 23.5 to 25.4	M1 A1 A1 3	or 44-46 and 68-70 incl. dep A1 Must look back, see method. No wking, ans in range: M1A1A1
(ii) 0.6 or 60% CF 720 63 to 64	M1 M1 A1 3	Seen or implied Seen or implied 55.5 to 56: SC B1
(iii) 1200 – 860 = 340	M1 A1 2	Allow 1200 – (850 to 890) 310 to 350
(iv) 340/1200 0.283^5 = 0.00183	M1 M1dep A1 3	their (iii)/1200 [their (iii)/1200]^5 exactly Allow 0.00114 to 0.00212 ≥ 2 sfs $^{340}C_5/^{1200}C_5$ M1
(v) Incorrect reason or ambiguity: B0B0. Otherwise: Too low, or should be 26 or 27 or 2 or 3 higher	B2 2	eg IQR = 55–35 = 20 or IQR = value >27 or new info' implies straight line: B1 or originally, majority in range 35 – 55 are at top of this range: B1
	13	
6 (i) $a={}^{4}/_{5}$, $b={}^{1}/_{5}$ $c={}^{1}/_{4}$, $d={}^{3}/_{4}$ $e={}^{3}/_{4}$, $f={}^{1}/_{4}$	B1 B1B1 B1 4	Or : B1 { ie: a, b : B1 B1 { another pair : B1B1 B1B1 { third pair : B1
(ii) ${}^{1}/_{2}$x ${}^{4}/_{5}$x${}^{1}/_{2}$ + ${}^{1}/_{2}$x${}^{1}/_{5}$x ${}^{1}/_{4}$ + ${}^{1}/_{2}$x ${}^{3}/_{5}$x ${}^{3}/_{4}$ $=\frac{9}{20}$ (**AG**) with no errors seen	M2 A1 3	M1: one correct product (M2 needs +) ft their values for M mks only
(iii) 1/10 + 9/20 + k + 1/5 = 1 oe or ${}^{1}/_{2}$x${}^{1}/_{5}$x${}^{3}/_{4}$ + ${}^{1}/_{2}$x${}^{3}/_{5}$x${}^{1}/_{4}$ + ${}^{1}/_{2}$x${}^{2}/_{5}$x${}^{1}/_{2}$ $k={}^{1}/_{4}$ oe	M1 A1 2	ft their values for M mk only
(iv) $\Sigma xp(x)$ $=1\frac{3}{4}$ oe $\Sigma x^2p(x)$ [$=3\frac{17}{20}$] $\Sigma x^2p(x)$ – (their μ)^2 63/80 or 0.788 (3 sfs)	M1 A1 M1 M1ind A1 5	Allow omit 1st term only. Not ISW, eg ÷ 4 cao Allow omit 1st term only. Not ISW, eg ÷ 4 Subtract (their μ)^2, if result +ve Follow their k for M mks only $\Sigma(x-\mu)^2p(x)$: Single consistent pair: M1 Rest correct : M1
	14	

7 (i) ${}^{18}C_7$ or ${}^{18!}/_{(11!\times 7!)}$	M1		
= 31824	A1	2	cao
(ii) ${}^5C_2 \times {}^6C_2 \times {}^7C_3$ or 5250	M2		M1: 1 correct ${}^nC_r$ or mult any three ${}^nC_r$s
÷ 31824	M1		Divide by their (i). Indep
= 875/5304 or 5250/31824 oe			If cancelled, must be clear have ÷ 31824
or 0.165 (3 sfs)	A1	4	
			5 x 4 x 6 x 5 x 7 x 6 x 5 x 7! / 18x17x 16 x 15 x 14 x 13 x 12 x $2!^2$x3!
			Correct 7 fractions mult: M1
			x7!: M1}
			÷ ($2!^2$x3!): M1}both dep any 7 fracts mult
(iii) 5 from W & 2 from (G + H)	M1		Seen or implied, eg by combs or list
${}^7C_5 \times {}^{11}C_2$ or 1155	M1		
÷ 31824	M1		Divide by their (i). Indep
= 385/10608 or 1155/31824 oe		4	
or 0.0363 (3 sfs)	A1		
			7 x 6 x 5 x 4 x 3 x 11 x 10 x 7! / 18x17x16 x 15 x14 x 13 x 12 x 5! x 2!
			Correct 7 fractions mult: M1
			x 7! : M1}
			÷ (5! x 2!): M1} both dep any 7 fracts mult
(iv) (2, 2, 3) or (2, 3, 2) or (3, 2, 2)	M1		Any one. Seen or implied eg by combs
${}^5C_2\times{}^6C_2\times{}^7C_3+{}^5C_2\times{}^6C_3\times{}^7C_2$			M1: one correct product.
$+{}^5C_3\times{}^6C_2\times{}^7C_2$	M2		NOT ${}^5C_2\times{}^6C_2\times{}^7C_2$
(÷ 31824)			(No mk for ÷ 31824)
= 175/442 or 12600/31824 oe			
or 0.396 (3 sfs)	A1	4	
			~~Equiv method; ((ii) + etc) can imply M mks~~
			5 x 4 x 6 x 5 x 7 x 6 x 7! / 18x17x16 x 15 x14 x 13 x $2!^2$x3!
			Correct 6 fractions mult: M1
			x 7! : M1}
			÷ ($2!^2$ x 3!) : M1}both dep any 6 fracts mult
			Complement method:
			Triple with total 7, incl at least one 0 or 1 or (0, 7) or (1, 6) seen or implied: M1
			One correct prod seen, eg ${}^5C_0x{}^6C_2x{}^7C_5$ M1
			Full correct method, incl "1 – " M1
	14		